Free and Second-Best Entry in Oligopolies with Network Effects

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Abstract

We compare the number of firms in equilibrium in a Cournot industry with positive network effects and complete compatibility, under free and second-best entry. Under free entry, the firms decide whether to enter the market or not; in the second-best problem, the number of firms is established by the regulator in order to maximize social welfare (the regulator controls entry but not production). We show that when individual equilibrium output decreases with entry (business-stealing competition), free entry may lead to more or less firms than the second-best problem. This contrasts with the standard (non-network) Cournot oligopoly model, wherein with business-stealing competition, free entry leads to an excessive number of firms compared to the second-best solution.

JEL codes: D43, L13, L14.

Keywords: Cournot competition, positive network effects, complete compatibility, under-entry, excessive entry.

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1 Introduction

In their seminal work on entry, Mankiw and Whinston [1986] compare the number of firms that enter a market under free entry against the second-best solution, that is, against the number of firms that a social planner chooses in order to maximize social welfare.[1] To this end, they consider a two-stage game where in the first stage, the number of firms is established by either free or regulated entry, and in the second one, there is Cournot competition. In any case (free or regulated entry), each firm must pay an entry cost.

Mankiw and Whinston [1986] find that if in the second stage of the game, individual equilibrium output decreases with entry (business-stealing effect), free entry leads, in general, to a larger number of firms.[2] Later on, Amir et al. [2014] showed that if the per-firm equilibrium output increases with entry (business-enhancing effect), the number of firms under free entry is lower than the second-best solution (that is, there is "under-entry"). Hence, in a standard Cournot oligopoly, excessive or under-entry crucially depends on whether competition is business-stealing or business-enhancing.

We introduce positive network effects to the entry problem studied by Mankiw and Whinston [1986] and Amir et al. [2014], and find that under-entry is more common in network industries. As it is the case in a non-network industry, under-entry in the presence of network effects is obtained when there is business-enhancing competition, but more interestingly, it can also be attained in the presence of business-stealing effects, which is a major reversal to the standard model with no network effects.

Another reason for under-entry to be more plausible in the presence of network effects is that the scope for the business-enhancing effect is broader in network industries. As deeply discussed by Amir [1996] and Amir and Lambson [2000], business-enhancing competition is rare in a standard Cournot oligopoly (with no network effects), and thus, under-entry is as well. For instance, if an inverse demand function is log-concave, competition is always

[1] In the second-best problem, the regulator controls only the number of firms in the market. In the first-best, the social planner controls both entry and output.

[2] Specifically, if n^e is the number of firms under free entry, and n^* is the second-best solution, Mankiw and Whinston [1986] show that $n^e \geq n^* - 1$ under business-stealing competition. That is, free entry might lead to a lower number of firms, but only by one firm. For simplicity, we refer to this result as "excessive entry".

business-stealing. Log-concavity is a condition satisfied by many inverse demand functions used in the literature, for example, by a linear inverse demand function. With network effects, there might be business-enhancing competition even if the inverse demand function is log-concave.

In particular, we consider oligopolies with positive network effects and complete compatibility. Markets with positive network effects are such that the consumers' willingness-to-pay increases with the number of buyers that are expected to purchase a compatible good (the expected size of the network). This effect is also known as "demand-side economies of scale". By complete compatibility, we mean that, as defined by Katz and Shapiro [1985], the goods are compatible no matter which firm produced them, and therefore, there is one single-network. Some examples are the telephone (landline and cell phones), some instant messaging applications, DVDs, fashion, and goods created under the umbrella of a standard-setting organization.[3]

Amir and Lazzati [2011] provide a thorough study of industries with network effects and complete compatibility. They analyze the important issue of viability (the existence of a non-zero equilibrium), and the effects of entry on the industry. In particular, they prove that per-firm equilibrium profits may increase or decrease with entry of new competitors.

The purpose of our paper is two-fold. First, we weaken the conditions in Amir and Lazzati [2011] that lead to decreasing or increasing individual equilibrium profits with respect to entry. Then we compare the second-best solution against the free entry outcome based on the comparative statics of entry on the resulting individual output. We focus on the case where per-firm profits decrease with entry since such is the case in a standard Cournot oligopoly (with no network effects); in addition, the results for increasing per-firm profits are trivial and thus, only briefly discussed.

As we mentioned before, we show that in the presence of network effects, the standard result of under-entry prevails when competition is business-enhancing, but under-entry is

[3] Nayyar [2004] studies the effects of entry in the long-distance telephone services market in the US following the forced divestiture of AT&T in 1984; he focuses on price dispersions and abstracts from incorporating network effects into the model. Some of the equilibrium consequences of standard-setting organizations are discussed in Shapiro [2000] and Samano and Santugini [2020].

also possible under business-stealing competition. Intuitively, if the network effect is sufficiently strong, it can offset the business-stealing effect, mimicking the results of the business-enhancing competition.

In the absence of network effects, and with business-stealing competition, consumers are better off with free entry than with a regulated industry, since consumer surplus is increasing in the number of firms and free entry leads lo a larger number of them. The opposite may happen in a network industry, since under-entry is possible, the consumers may prefer the industry to be regulated.

Some related studies are Gama [2019] and Woo [2013]. The former conducts a similar analysis on endogenous entry in network industries, but instead of studying industries with a single-network (like this paper), Gama [2019] considers industries with firm-specific (incompatible) networks. In that case, the results on entry are aligned with those of a non-network industry.[4] Such study is based on the setting by Amir et al. [2019], and the comparative statics analysis in Gama et al. [2020].

Woo [2013] compares free versus socially optimal entry in an industry with status effects of consumption, that is, when agents consider others' consumption to determine if theirs is enough to maintain or improve their social status. Woo shows that under sufficiently strong status effects, there is excessive entry, and his approach differs from ours in that his analysis is based on the utility function rather than on the demand function.[5]

Another work concerned with social welfare and network effects is Guimaraes et al. [2020]. The authors review efficiency in dynamic coordination games with timing frictions and includes applications in network industries. One takeaway is that it is actually efficient that the agents stay in a low-quality network when the social transaction costs of switching to a higher-quality network are larger than the social future gains. Such is the case of the QWERTY keyboard, which is intrinsically worse than the Dvorak alternative, but potentially preferred by the social planner. A similar argument can be made for Windows over Linux,

[4]With firm-specific networks, individual equilibrium output always decreases with entry, and under-entry may hold but only by one firm.

[5]Other (recent) studies that consider endogenous entry are Suzuki [2020] and Schröder and Sørensen [2020]. The first one with innovation in a dynamic general equilibrium setting, the second one in a model of monopolistic competition with endogenous quality.

see Guimaraes and Pereira [2016] for more details.

The next section provides the setting of the model and our assumptions, Section 3 presents the results, and Section 4 concludes. In the Appendix we present the proofs to our results.

2 The Model

Before specifying the two entry games under study, we describe the oligopoly model with network effects and the equilibrium concept since they are not standard in the literature. To do so, we follow the setup in Amir and Lazzati [2011].

We consider an oligopoly with n identical firms and positive network effects, in the sense that a consumer's willingness-to-pay increases with the number of people purchasing a compatible good. There is also complete compatibility, that is, the firms produce homogenous goods that are perfectly compatible with each other, therefore, there is a single-network conformed by all the goods in the industry. Given the positive externalities in demand, consumers form a belief on the size of the network, the "expected size of the network", that cannot be directly influenced by the firms.

Since the firms are identical, all of them face the same inverse demand function $P(z, s)$, where z denotes total output, and s is the expected size of the network. Every consumer buys at most one unit of the good, therefore, the expected size of the network is equivalent to the expected number of buyers in the industry. For simplicity, we assume that production is costless.

By symmetry, the profit of any given firm is $\pi(x, y, s) = xP(x + y, s)$, where x is the individual output of the firm and y denotes the joint output by the other $(n-1)$ firms, that is, $z = x + y$. Then, each firm solves

$$\max_{x \geq 0} \pi(x, y, s). \tag{1}$$

As we said earlier, s is an exogenous parameter for the firm, since the latter cannot affect the consumers' expectations about the size of the network.

The equilibrium concept for this model is defined as follows. It is due to Katz and Shapiro [1985] and it is called *fulfilled or rational expectations Cournot equilibrium* (henceforth RECE or simply "equilibrium").

Definition 1 *A RECE consists of a vector of individual outputs* $(x_1^*, x_2^*, ..., x_n^*)$ *and an expected network size* s *such that:*
1. $x_i^* \in \arg\max\{xP(x + \sum_{j\neq i} x_j^*, s) - K : x \geq 0\}$ *for* $i = 1, 2, \ldots, n$*; and*
2. $\sum_{i=1}^n x_i^* = s$.

The RECE establishes that in equilibrium, both the consumers and the firms correctly predict the market outcome, and the expectations of the consumers are fulfilled. With complete compatibility, there is only one single-network composed by all the consumers of the compatible products. Although the firms compete in quantity, they cannot influence the consumers' expectations of the network size, so that they are "network-size taking". The firms work together to build a common network, yet, they compete with each other to serve it.

To guarantee the existence of a symmetric RECE, we make the following assumptions. Besides ensuring that at least one symmetric equilibrium exists, Assumptions (A1)-(A3) imply that no asymmetric equilibria exist (Amir and Lazzati [2011], Theorem 2). The subindices of $P(z, s)$ denote partial derivatives.

(A1) $P : \mathbb{R}^+ \times \mathbb{R}^+ \to \mathbb{R}^+$ is twice continuously differentiable, $P_1(z, s) < 0$ and $P_2(z, s) > 0$.

(A2) Each firm's individual output is bounded by $X > 0$.

(A3) $P(z, s)P_{12}(z, s) - P_1(z, s)P_2(z, s) > 0$, for all (z, s).

In particular, $P_1 < 0$ reflects the law of demand, and $P_2 > 0$ the demand-side economies of scale (willingness-to-pay increases with the expected size of the network). The firms' capacity constraint imposed by (A2) is a technical requirement that guarantees the existence of equilibria and prevents technical problems with unbounded output, but the results do not depend on the magnitude of X.

Assumption (A3) implies that the elasticity of demand increases with the expected size of the network, which is a characteristic of industries with network effects.[6] A consumer's reaction to a price change is reinforced by the reaction of the rest of the consumers, which moves in the same direction.

[6] Specifically, (A3) implies that $\frac{\partial \epsilon}{\partial s} \geq 0$, where $\epsilon = -\left[\frac{\partial P(z,s)}{\partial z} \frac{z}{P(z,s)}\right]^{-1}$ is the elasticity of demand.

Since the equilibria are symmetric and for ease in the notation, we will denote the equilibrium variables with the sub-index n, whenever there are n identical firms. That is, x_n denotes the per-firm equilibrium output and $z_n = nx_n$ the equilibrium industry output; π_n and W_n are the corresponding individual profits and social welfare, respectively.

Now we turn to the description of the (two) two-stage games under consideration, where in each of them we consider the subgame-perfect equilibria. The first two-stage game consists of a free entry game and firms make decisions in the two stages. In the first stage, firms decide whether to enter the market or not, if they decide to enter, they pay the entry cost $K > 0$. In the second stage, the firms compete in quantity according to the model described above.

If π_n (per-firm equilibrium profit in the second stage) decreases with n, the number of firms under free entry is $n^e \in \mathbb{N}$ such that

$$\pi_{n^e} - K \geq 0 \text{ and } \pi_{n^e+1} - K < 0.$$ [7]

The second two-stage game under consideration is the second-best problem, where the industry is regulated. In this game, the regulator first establishes the number of firms by maximizing social welfare; to enter the market, a firm must pay the entry cost $K > 0$. In the second stage, the firms compete in quantity according to the model described in the first part of this section, the oligopoly model with network effects and complete compatibility.

Therefore, the second-best solution is $n^* \in \mathbb{N}$ that solves the regulator's problem

$$\max_n \{W_n - nK\}, \tag{2}$$

where W_n is the social welfare corresponding to the equilibrium in the second stage of the game.

Now that we have presented the two games, we state a final assumption in addition to (A1)-(A3) to guarantee that at least one firm survives in the market in both games.

(A4) $\pi_1 > K$, $K > 0$.

In the next Section we characterize the properties of the equilibria of the (two) two-stage games introduced here. Our goal is to understand the entry outcomes of the two

[7] Notice that π_n and W_n (below) do not include the entry cost $K > 0$. In this sense, our notation is different from that in Mankiw and Whinston [1986], but in line with Amir et al. [2014].

games and how they compare to each other. For a better discussion of our results, it is important to keep in mind that in a standard (non-network) Cournot oligopoly with costless production, per-firm equilibrium profits always decrease with the number of firms (Amir and Lambson [2000], Theorem 2.2-c). In addition, whether there is excessive or under-entry in the standard Cournot oligopoly depends on whether there are business-stealing or business-enhancing effects, respectively (Amir et al. [2014], Proposition 1). Specifically, with business-stealing competition, there might be under-entry, but only by one firm ($n^e \geq n^* - 1$), otherwise, $n^e \leq n^*$ with business-enhancing competition. Our results show that this is no longer the case when there are network effects.

3 Results

In the absence of network effects, excessive entry is a direct consequence of the business-stealing effect (Mankiw and Whinston [1986] and Amir et al. [2014]), that is, if individual output decreases with competition, free entry leads to a larger number of firms than in the second-best. Under business-stealing competition, the firms have the incentive to enter the market by stealing existing firms' clients until their profits become zero; even though total output increases, industry profits are reduced to zero, which hurts social welfare. The regulator internalizes this fact and allows a lower number of firms into the market.

With network effects and complete compatibility, entry always increases total output (see Theorem 1-i below), and thus the size of the network. Therefore, even if entry reduces individual output, it also creates a social gain by expanding the network, which might lead to a larger number of firms under the second-best problem (under-entry).

For a better understanding of this outcome, consider the problem of the regulator (2):

$$\max_n \left[W_n - nK = \int_0^{z_n} P(t, z_n)dt - nK \right];$$

ignoring the integer constraint and assuming that W_n is differentiable, the optimal number of firms n^* satisfies the first order condition

$$\frac{dW_{n^*}}{dn} - K = \pi_{n^*} - K + n^* P(z_{n^*}, z_{n^*}) \frac{dx_{n^*}}{dn} + \frac{dz_{n^*}}{dn} \int_0^{z_{n^*}} P_2(t, z_{n^*})dt = 0. \qquad (3)$$

Under business-stealing competition, we have $\frac{dx_n}{dn} \leq 0$ for all n, and by Amir and Lazzati [2011] (see Theorem 1-i below), total output increases with entry, $\frac{dz_n}{dn} \geq 0$ for all n. Provided the positive network externality, $P_2 > 0$, the sign of

$$n^* P(z_{n^*}, z_{n^*}) \frac{dx_{n^*}}{dn} + \frac{dz_{n^*}}{dn} \int_0^{z_{n^*}} P_2(t, z_{n^*}) dt \tag{4}$$

is ambiguous, and therefore, $\pi_{n^*} - K$ might be positive or negative.

If expression (4) is negative, then $\pi_{n^*} - K$ must be positive in order to satisfy the first order condition (3). Therefore, free entry leads to at least n^* firms, which is what we call excessive-entry. On the other hand, if expression (4) is positive, there is under-entry. Consequently, the comparison between free and second-best entry cannot be predicted in general, and it depends on the primitives of the industry.

Observe that in the absence of network effects, the second term in expression (4) becomes zero and hence, the remaining term is negative, leading to excessive entry. As mentioned before, in a standard Cournot oligopoly, profits always decrease with n and excessive/under-entry crucially relies on whether individual output decreases or increases with entry.

To present our results in an organized manner, we first state sufficient conditions for the comparative statics of individual profits and output with respect to entry. These comparative static results refer to the RECE, that is, to the equilibrium in the second stage of the games under comparison (the oligopoly model with network effects). Recall that such equilibrium is sub-indexed by n whenever there are n firms in the market. Since uniqueness of the equilibrium is not guaranteed, all of our results refer to the extremal equilibria, that is, to the minimal and maximal equilibria.

Consider the following functions:

$$\begin{aligned}
\Delta_1(z) &\equiv P_1(z,z) + P_2(z,z), \\
\Delta_2(z) &\equiv P(z,z)[P_{11}(z,z) + P_{12}(z,z)] - P_1(z,z)\Delta_1(z), \text{ and} \\
\Delta_3(z) &\equiv -P_1(z,z)\Delta_1(z) + \Delta_2(z).
\end{aligned}$$

Amir and Lazzati [2011] introduce $\Delta_1(z)$ and $\Delta_2(z)$ and use them to conduct comparative statics of equilibrium per-firm output (x_n) and profits (π_n). In particular, they show that

$x_n \geq x_{n+1}$ ($x_n \leq x_{n+1}$) whenever $\Delta_2(z) \leq 0$ ($\Delta_2(z) \geq 0$) on $[z_n, z_{n+1}]$. If both $\Delta_1(z) \leq 0$ and $\Delta_2(z) \leq 0$ ($\Delta_1(z) \geq 0$ and $\Delta_2(z) \geq 0$), per-firm profits decrease (increase) in n. Total output always increases in n. We summarize these results in Lemma 1 and Theorem 1, since they are key in discussing the results of this paper. The proofs are available at the original reference.

Lemma 1 *(Amir and Lazzati [2011], Lemma 9) At any interior equilibrium*
i) $x_n \geq x_{n+1}$ *if* $\Delta_2(z) \leq 0$ *on* $[z_n, z_{n+1}]$*;*
ii) $x_n \leq x_{n+1}$ *if* $\Delta_2(z) \geq 0$ *on* $[z_n, z_{n+1}]$.

Theorem 1 *(Amir and Lazzati [2011], Theorems 8-i and 10) At any interior equilibrium*
i) $z_n \leq z_{n+1}$ *for all* n*, and*
ii) $\pi_n \geq \pi_{n+1}$ ($\pi_n \leq \pi_{n+1}$) *if* $\Delta_1(z) \leq 0$ *and* $\Delta_2(z) \leq 0$ ($\Delta_1(z) \geq 0$ *and* $\Delta_2(z) \geq 0$) *on* $[z_n, z_{n+1}]$.

Notice that $\Delta_1(z)$ is the derivative of $P(z, z)$ with respect to z, in other words, $\Delta_1(z)$ accounts for the total change in price when aggregate output changes along the fulfilled expectation path. Then, the effects of entry on the equilibrium price are given by $\Delta_1(z)$. Specifically, since total output increases with entry (Theorem 1-i), $p_n \geq p_{n+1}$ ($p_n \leq p_{n+1}$) if $\Delta_1(z) \leq 0$ ($\Delta_1(z) \geq 0$) on $[z_n, z_{n+1}]$, where p_n denotes the corresponding equilibrium price with n firms.

The interpretation of $\Delta_2(z)$ is less straightforward. As we explain next, $\Delta_2(z)$ refers to the fact that the scope for business-enhancing competition is broader in the oligopoly with network effects. Observe that $\Delta_2(z)$ can be rewritten as $\Delta_2 = [P(z, z)P_{12}(z, z) - P_1(z, z)P_2(z, z)] + [P(z, z)P_{11}(z, z) - P_1^2(z, z)]$, the first term is strictly positive by (A3) and the second one is positive whenever $P(z, s)$ is log-convex in z. In the absence of network effects, log-convexity of the inverse demand function is a sufficient condition to have business-enhancing competition (Amir and Lambson [2000], Theorem 2.4), which is stronger than having $\Delta_2(z) \geq 0$, sufficient to have business-enhancing competition with network effects (Lemma 1-ii).

The novel function in this paper is $\Delta_3(z)$, which predicts whether per-firm profits increase or decrease with n. Recall that in standard Cournot, per-firm profits always decrease with n and thus, no analogous condition is needed. Lemma 2 establishes the relationship between

$\Delta_3(z)$ and π_n in our setting with a single-network. This and the rest of the proofs can be found in Section 4.

Lemma 2 *At any interior equilibrium of the second stage*
i) $\pi_n \geq \pi_{n+1}$ *if* $\Delta_3(z) \leq 0$ *on* $[z_n, z_{n+1}]$*;*
ii) $\pi_n \leq \pi_{n+1}$ *if* $\Delta_3(z) \geq 0$ *on* $[z_n, z_{n+1}]$.

Since $\Delta_3(z) = -P_1(z,z)\Delta_1(z) + \Delta_2(z)$, Lemma 2 provides a weaker condition than that in Theorem 1-*ii*. To see this, note that $\Delta_1(z) \leq 0$ and $\Delta_2(z) \leq 0$ imply that $\Delta_3(z) \leq 0$, but the condition $\Delta_3(z) \leq 0$ allows for the possibility that $\Delta_1(z)$ and $\Delta_2(z)$ have different signs. For instance, it could be that $\Delta_1(z) \geq 0$, $\Delta_2(z) \leq 0$ and $\Delta_3(z) \leq 0$, that is, per-firm output decreases with entry ($\Delta_2(z) \leq 0$) and the network effect is stronger than the market effect ($\Delta_1(z) \geq 0$), but not strong enough to increase the individual profits ($\Delta_3(z) \leq 0$). Conditions $\Delta_1(z) \leq 0$ and $\Delta_2(z) \leq 0$ are aligned in the sense that both the equilibrium price and per-firm output decrease with entry, thus leading to decreasing per-firm profits. The comparison of the condition $\Delta_1(z) \geq 0$ and $\Delta_2(z) \geq 0$ with $\Delta_3(z) \geq 0$ is analogous.

To clarify the relevance of Lemma 2, we provide the following example, which is based on Example 1 of Amir and Lazzati [2011].[8]

Example 1. Consider an industry with positive network effects, complete compatibility, and inverse demand function $P(z,s) = \exp\left(-\frac{z}{\exp(1-1/s)}\right)$. Given s, a firm solves

$$\max_x \left[x \exp\left(-\frac{x+y}{\exp(1-1/s)}\right) - K \right]$$

in the second-stage of the game, which leads to the best-response $x(y,s) = \exp(1-1/s)$. The symmetric RECE is then given by the fixed point of $f(s) = n\exp(1-1/s)$, that is, the RECE is implicitly given by $z_n = n\exp(1-1/z_n)$ and thus, $\pi_n = x_n \exp(-n)$, with $x_n = z_n/n$.

First observe that when $n = 1$, the unique non-zero RECE is $z_1 = x_1 = 1$. Since z_n increases with n (Theorem 1-*i*), we have that $z_n \geq 1$ for all $n \geq 1$, a result that will become useful further below.[9]

[8] In their example, the inverse demand function is $P(z,s) = \exp\left(-\frac{2z}{\exp(1-1/s)}\right)$.

[9] Recall that the comparative statics results in this paper hold for the minimal and maximal equilibria. In this example, we focus on the largest one since the lowest is the trivial equilibrium, $z_n = 0$, for all $n \geq 1$.

After some calculations we get

$$\Delta_1(z) = -P_1(z,z)(1-z)/z$$

and

$$\Delta_2(z) = -P_1(z,z)P(z,z)/z^2.$$

Since $P_1 < 0$, we have that $\Delta_2(z) \geq 0$ for all $z > 0$. Provided that $z_n \geq 1$ for all n, Lemma 1 predicts that x_n globally increases with respect to n. Nonetheless, Theorem 1-*ii* is silent on predicting the direction of change of π_n. This is because $\Delta_1(z) \leq 0$ and $\Delta_2(z) \geq 0$ for all $z \geq 1$, and Theorem 1-*ii* requires equal signs to achieve a conclusion.

To see that π_n decreases with entry, we will use Lemma 2. To this end, note that

$$\Delta_3(z) = \frac{-P_1(z,z)P(z,z)}{z^2}\left[1 - \frac{z(z-1)}{\exp(1-1/z)}\right] \leq 0$$

for all $z \geq 1.8548$.[10] But $z_2 = 4.311$, then, $\Delta_3(z) \leq 0$ for all $z \geq z_2 = 4.311$, and by Lemma 2, π_n decreases for all $n \geq 2$ (recall that z_n increases in n). To see that π_n globally decreases in n, the reader can easily verify that $\pi_1 = 0.3679 > \pi_2 = 0.2917$.

Finally, observe that $\Delta_3(z_1) = \Delta_3(1) = -P_1(1,1)P(1,1) = \exp\{-2\} \geq 0$, which shows that $\Delta_3(z) \leq 0$ for all $z \geq z_1 = 1$ is only a sufficient, but not necessary, condition to have π_n globally decreasing in n.

Next, we present our results on endogenous entry in network industries. They are classified based on characteristics of the endogenous variables π_n and x_n. The conditions on such variables can be replaced by the conditions on $P(z,s)$ given by Lemmas 1 and 2, but since the latter provides only sufficient (but not necessary) conditions, we write the conditions in general to broaden the applicability of our results.

3.1 Endogenous entry

According to Lemma 2, individual equilibrium profits might increase or decrease with entry. We focus on the case where they globally decrease, $\pi_n \geq \pi_{n+1}$ for all n, because

[10] This can be obtained from the roots of $1 - \frac{z(z-1)}{\exp(1-1/z)} = 0$, and given $P_1 < 0$.

such is the case in the standard Cournot model.[11] In this context we establish an important difference between the industries with and without network effects: with business-stealing competition, under-entry (by more than one firm) may occur in the presence of network effects, which does not happen otherwise.

3.1.1 Business-stealing competition

The industry studied in this section is attained in a global sense (per-firm output and per-firm profits globally decreasing in n) whenever $\Delta_2(z) \leq 0$ and $\Delta_3(z) \leq 0$ for all $z \geq z_1$ (Lemmas 1 and 2). Although this kind of industry is typical, it is the hardest to predict in a general way. As we explained before (with the help of equation (4)) there might be excessive or under-entry when x_n and π_n decrease with n. It would be ideal to provide conditions on $P(z, s)$ so that we can predict the sign of equation (4), but this is not an easy task since it is hard to measure the magnitude of the opposite effects.

Yet, as we show next, it is easy to characterize an industry with a linear inverse demand function, in both s and z, and show that a sufficiently strong network effect leads to under-entry in the presence of business-stealing competition.

Example 2. Consider an industry with a compatible network and inverse demand function given by $P(z, s) = a + bs - z$, with $a > 0$, $0 < b < 1$, and $0 < K < a^2/(2-b)^2$. The constraint on K guarantees that at least one firm survives in the market (Assumption (A4)). Given s, the reader can easily verify that the best reply of any firm is

$$x(y, s) = \frac{a + bs - y}{2},$$

and the symmetric RECE (z_n) is the fixed point of $f(s) = n\frac{a+bs}{n+1}$. Hence, total output becomes

$$z_n = \frac{na}{1 + n(1-b)},$$

[11] Moreover, when individual profits increase, the results are trivial. Free entry leads to an infinite number of firms, since the firms are better off when there are more of them in the market. Similarly, it is easy to show that if individual output also increases with entry, then social welfare increases with n and the second-best solution also consists of allowing an infinite number of firms.

and per-firm output and profits are given by $x_n = \frac{a}{1+n(1-b)}$ and $\pi_n = \frac{a^2}{[1+n(1-b)]^2}$, respectively. As predicted by Lemmas 1 and 2, x_n and π_n decrease with n,[12] and whether there is excessive or under-entry depends only on the magnitude of the network effect measured by b. In particular, we have (the calculations are shown in the Appendix)

i) If $b < 0.5$, there is excessive entry, $n^e \geq n^*$,

ii) If $b \geq 0.5$, there is under-entry, $n^e \leq n^*$.

The parameter b measures the sensitivity of the consumers' willingness-to-pay to the expected size of the network and it is the only determinant of whether there is excessive or under-entry. When the sensitivity is low ($b < 0.5$), the business-stealing effect is stronger than the network effect (see equation (4)), and more firms than the socially optimal level decide to enter. On the other hand, if b is relatively large ($b \geq 0.5$), the network effect prevails and there is under-entry.

The previous finding establishes a clear difference between industries with and without network effects. In the absence of network effects, under-entry may happen but only by one firm (without network effects, $n^e \geq n^* - 1$; see Amir et al. [2014], Proposition 1). With network effects, under-entry by more than one firm is possible, for example, if we set $a = 8$, $K = 4$, and $b = 0.7$ in Example 2, free entry yields a total of six fewer firms than in the second-best solution, $n^e = 10 < 16 = n^*$. This departure from the standard Cournot oligopoly is due to the network effect depicted by b.

An important consequence of Example 2 is that consumers may be better off with a regulator than under free entry. In the absence of network effects, consumer surplus increases with the number of firms, consequently, consumers prefer the free entry scheme which leads to more firms.[13] In the industry considered in Example 2, consumer surplus also increases with the number of firms (Amir and Lazzati [2011], Theorem 11-i), hence, when $b \geq 0.5$, consumers prefer to be in a regulated industry.[14]

[12] Note that $\Delta_1(z) = \Delta_2(z) = b-1 < 0$ and $\Delta_3(z) = 2(b-1) < 0$ when $0 < b < 1$. If $b > 1$, we get opposite signs and x_n and π_n increase with respect to n. If $b = 1$, $x_n = a$ and $\pi_n = a^2$ are invariant to entry, in line with Lemmas 1 and 2.

[13] Except for the case $n^e = n^* - 1$.

[14] Specifically, Theorem 11-i in Amir and Lazzati [2011] establishes that consumer surplus increases in n,

3.1.2 Business-enhancing competition

When individual profits decrease with competition but individual output increases, the result in the industry with network effects is analogous to that of the non-network industry: there is under-entry. With network effects and business-enhancing competition, both individual and total output have the same direction of change and therefore, there is always under-entry. Specifically, equation (4) is positive whenever x_n increases in n (we include the proof considering that n is an integer).

Recall that if $\Delta_2(z) \geq 0$ on $[z_{n^*}, z_{n^*+1}]$ and $\Delta_3(z) \leq 0$ for all $z \geq z_1$ (Lemmas 1 and 2), we have the hypothesis in the next result.

Proposition 1 *At any interior equilibrium, if $\pi_n \geq \pi_{n+1}$ for all n, and $x_{n^*} \leq x_{n^*+1}$, then, $n^e \leq n^*$.*

Proposition 1 states that whenever per-firm profits globally decrease in n and per-firm output increases at n^*, free-entry leads to a lower number of firms, compared to the second-best solution. The industries with and without network effects share the characteristic of under-entry with business-enhancing competition, but as we discussed earlier (when introducing $\Delta_2(z)$), this type of competition is more plausible in industries with network effects (Amir and Lazzati [2011]).

For instance, in a non-network industry, business-enhancing competition can be attained for all n only in the absence of variable costs. Similarly, when $P(z, s)$ is log-concave in z, business-enhancing competition is impossible in the standard Cournot oligopoly (see Amir [1996] and Amir and Lambson [2000] for more details), but it is feasible with network effects, as shown by Example 1.

In such example, $P(z, s) = \exp\left(-\frac{z}{\exp(1-1/s)}\right)$ is log-linear in z, and thus, log-concave in z, yet, per-firm output globally increases in n (business-enhancing effect). Moreover, per-firm profits globally decrease in n and as predicted by Proposition 1, there is under-entry. For instance, if $K = 0.05$, we have that $n^e = 3 < n^* = 7$.

$CS_n \leq CS_{n+1}$, if $\Delta_1(z) \leq 0$ on $[z_n, z_{n+1}]$ or $P_{12}(z, s) \leq 0$ for all z, s. Both conditions are satisfied by Example 2, since $\Delta_1(z) = b - 1 < 0$, by $b < 1$, and $P_{12}(z, s) = 0$.

4 Final Remarks

We have shown that introducing network effects to the standard Cournot oligopoly model, as in Amir and Lazzati [2011], changes the conventional wisdom that excessive entry (under-entry) is a direct consequence of the business-stealing (business-enhancing) effect. With a single-network, under-entry occurs under business-enhancing competition, but may also happen with the business-stealing effect. Moreover, business-enhancing competition is more plausible with network effects, then, under-entry in oligopolies may be more common than one would think.

Although per-firm equilibrium profits might increase with entry in the presence of network effects, we have focused on the case where per-firm profits decrease because such is the outcome in a regular (non-network) industry. Besides, when per-firm equilibrium profits increase, the results on entry are trivial: since the firms increase their profits with a new competitor, free entry leads to an infinite number of firms. If in addition, individual output also increases, it is easy to show that social welfare increases with entry and hence, the regulator will also allow an infinite number of firms. This environment with x_n and π_n increasing in n is easily obtained, for instance, when $b > 1$ in Example 2 (see Footnote 12).

Without network effects, business-stealing competition is the norm, and thus, excessive entry is as well. Consequently, the firms obtain lower profits under free entry than in a regulated industry, and the consumers benefit from it by being offered lower prices. With the existence of network externalities, there might be under-entry even in the presence of the business-stealing effect, then, the firms are better off with free entry, but the consumers are worse off. Therefore, one has to be very careful when allowing free entry of firms if the objective is to maximize consumer surplus; although it is natural to assume that free entry benefits consumers, it might be the opposite in the presence of network effects.

Appendix

Proof of Lemma 2.

First note that any interior equilibrium satisfies the first order condition at the second stage of the game

$$P(z_n, z_n) + x_n P_1(z_n, z_n) = 0,$$

which implies that $x_n = \frac{P(z_n,z_n)}{-P_1(z_n,z_n)}$. Taking the derivative with respect to n, we have

$$\begin{aligned}\frac{dx_n}{dn} &= \frac{P_1(z_n, z_n) + P_2(z_n, z_n) + x_n[P_{11}(z_n, z_n) + P_{12}(z_n, z_n)]}{-P_1(z_n, z_n)}\frac{dz_n}{dn} \\ &= \frac{\Delta_2(z_n)}{P_1^2(z_n, z_n)}\frac{dz_n}{dn}.\end{aligned}$$

Per-firm equilibrium profits are $\pi_n = x_n P(z_n, z_n)$. Taking the derivative with respect to n, substituting x_n and $\frac{dx_n}{dn}$, and rearranging terms, we have

$$\begin{aligned}\frac{d\pi_n}{dn} &= x_n[P_1(z_n, z_n) + P_2(z_n, z_n)]\frac{dz_n}{dn} + P(z_n, z_n)\frac{dx_n}{dn} \\ &= x_n\left(\Delta_1(z_n) + \frac{\Delta_2(z_n)}{-P_1(z_n, z_n)}\right)\frac{dz_n}{dn} \\ &= x_n\frac{\Delta_3(z_n)}{-P_1(z_n, z_n)}\frac{dz_n}{dn}.\end{aligned}$$

By $P_1 < 0$ and $\frac{dz_n}{dn} \geq 0$, the results follow. □

Results i) and ii) from Example 2.

The assumption $0 < K < a^2/(2-b)^2$ implies that at least one firm is active, $\pi_1 > K$.

In general, the problem of the regulator is

$$\max_n [W_n - nK = \int_0^{z_n} P(t, z_n)dt - nK],$$

with first order condition:

$$\pi_n - K + nP(z_n, z_n)\frac{dx_n}{dn} + \frac{dz_n}{dn}\int_0^{z_n} P_2(t, z_n)dt = 0. \tag{5}$$

When $P(z, s) = a + bs - z$, the left-hand side of equation (5) becomes

$$\frac{a^2(1+bn)}{(1+n(1-b))^3} - K = \frac{\pi_n(1+bn)}{1+n(1-b)} - K, \tag{6}$$

and the second order condition (SOC) is

$$\frac{a^2(4b-3-2bn(1-b))}{(1+n(1-b))^4} < 0,$$

which holds for $b \leq 0.82$ or for n sufficiently large, $n > (4b-3)/[2b(1-b)]$.

(i) Assume that $0 < b < 0.5$. At n^e+1, equation (6) becomes

$$\frac{\pi_{n^e+1}(1+b(n^e+1))}{1+(n^e+1)(1-b)} - K < \frac{K(1+b(n^e+1))}{1+(n^e+1)(1-b)} - K = \frac{K(n^e+1)(2b-1)}{1+(n^e+1)(1-b)} < 0.$$

The first inequality follows by the free entry condition, $\pi_{n^e+1} < K$, and the second one, by $0 < b < 0.5$. Then, provided that $W_n - nK$ is strictly concave in n (given $b \leq 0.82$), the regulator must decrease the number of firms in order to optimize social welfare, $n^* < n^e + 1$, which implies $n^* \leq n^e$.

(ii) Now suppose that $0.5 \leq b < 1$. Analogous to part (i), but using the fact that $\pi_{n^e} \geq K$, we have

$$\frac{\pi_{n^e}(1+bn^e)}{1+n^e(1-b)} - K \geq \frac{K(1+bn^e)}{1+n^e(1-b)} - K = \frac{Kn^e(2b-1)}{1+n^e(1-b)} \geq 0.$$

If $0.5 \leq b \leq 0.82$, the result follows immediately by the strict concavity of $W_n - nK$. Otherwise, if $0.82 < b < 1$, the second order condition holds when $n > (4b-3)/[2b(1-b)]$. Similarly, note that when $n = 1$, the regulator can (weakly) improve social welfare with an additional firm, which can be seen from equation (6):

$$\frac{\pi_1(1+b)}{2-b} - K > \frac{K(1+b)}{2-b} - K = \frac{K(2b-1)}{2-b} \geq 0$$

(the first inequality follows from (A4), $\pi_1 > K$, and the second one, from $0.5 \leq b < 1$). Hence, social welfare is first increasing and convex, then changes curvature and finally decreases. Altogether, we have $n^e \leq n^*$. □

Proof of Proposition 1.

First notice that since $P_1 < 0$ and $x_{n^*+1} > 0$ (if $x_{n^*+1} = 0$ the result is immediate, $n^* = n^e = 0$),

$$x_{n^*+1}P(z_{n^*+1}, z_{n^*+1}) < \int_0^{z_{n^*+1}} P(t, z_{n^*+1})dt - \int_0^{n^* x_{n^*+1}} P(t, z_{n^*+1})dt. \tag{7}$$

Also, by optimality of n^*,

$$W_{n^*} - n^*K \geq W_{n^*+1} - (n^*+1)K,$$

that is,

$$\int_0^{z_{n^*}} P(t, z_{n^*})dt - n^*K \geq \int_0^{z_{n^*+1}} P(t, z_{n^*+1})dt - (n^*+1)K. \tag{8}$$

Then, we have the following expressions

$$\begin{aligned}
&\pi_{n^*+1} - K \\
&\quad \leq \; x_{n^*+1}P(z_{n^*+1}, z_{n^*+1}) + \int_0^{z_{n^*}} P(t, z_{n^*})dt - \int_0^{z_{n^*+1}} P(t, z_{n^*+1})dt \\
&\quad \leq \; x_{n^*+1}P(z_{n^*+1}, z_{n^*+1}) + \int_0^{z_{n^*}} P(t, z_{n^*+1})dt - \int_0^{z_{n^*+1}} P(t, z_{n^*+1})dt \\
&\quad < \; \int_0^{z_{n^*}} P(t, z_{n^*+1})dt - \int_0^{n^*x_{n^*+1}} P(t, z_{n^*+1})dt \\
&\quad \leq \; 0.
\end{aligned}$$

The first inequality follows from $\pi_{n^*+1} = x_{n^*+1}P(z_{n^*+1}, z_{n^*+1})$ and (8); the second one from $z_{n^*+1} \geq z_{n^*}$ and $P_2 > 0$, and the third one from inequality (7). The last inequality is given by $n^*x_{n^*+1} \geq n^*x_{n^*} = z_{n^*}$, provided $x_{n^*+1} \geq x_{n^*}$.

Hence, $\pi_{n^*+1} < K$, and by $\pi_n \geq \pi_{n+1}$, we have the result. □

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